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# DESCRIPTION OF THE CREEP PROCESS AND LONG-TERM STRENGTH BY EQUATIONS WITH ONE SCALAR DAMAGE PARAMETER

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In the example of titanium and aluminum alloys we illustrate the possibility of describing the process of creep and fracture of materials whose strain in fracture depends on the magnitude and type of stressed state. The reduction of the experimental creep diagrams to a common curve in relative coordinates $\{\omega = A/A_*,\ \tau = t/t_*\}$ (A and t are the instantaneous values of the scattered energy and time, and $A_*$ and $t_*$ are the corresponding values in fracture) permits the damage equation to be written in the form of an equation of state $d\omega/dt = \varphi(\sigma)\psi(\omega)$, and the damage parameter to be related to experimentally measurable quantities. This capability removes the leeway (uncertainty) from the determination of the coefficients of the creep and damage equations and provides a unified procedure for obtaining them.

The consistency of an equation of state with one scalar parameter for describing the description of damage kinetics is substantiated experimentally in steady (smoothly varied with time) and stepped loading regimes. It is established that the cumulative damage in tension and compression differs for materials with different creep resistances, and the domain of validity of the governing equations is indicated.

The description of the prefracture creep process with allowance for damage of the material is utilized at the present time not only in long-term strength studies, but also in technological problems of morphogenesis in creep regimes and superplasticity in estimating the elapsed service life of a material in the product manufacturing stage. The description of material deformation processes from a unified point of view in the presence of technological creep (minutes to hours) and in subsequent service (tens to hundreds of thousands of hours) for assessment of the remaining high-test life of structural members poses a timely problem.

Investigations of creep in materials with simultaneous cumulative damage have been reported in many papers [1, 2], in which attempts have been made to coordinate the problem of determining stress–strain state with the problem of determining time to fracture by introducing a damage parameter based on phenomenological principles. Damage appears to be a tensor phenomenon in the most general case [3, 4], but certain difficulties are encountered in the practical application of a damage tensor. Attempts are currently underway to define the damage proneness of a material as a vector quantity [5], governing equations are being formulated with two kinetic damage parameters [6] or with a single parameter and different fracture criteria [7], and models utilizing the concept of a damage-prone continuum based on the model of a two-phase medium are being discussed [8], but equations with a single damage parameter are used by and large for investigations both in the domain of short-term creep and in the domain of long-term strength.

Rabotnov's model [9] is used to describe the deformation process of metallic materials in the majority of papers, where the investigated state of a material is characterized by a damage parameter ("fissility") $0 \le q \le 1$, along with various modifications based on the notion of effective stress in the form $\sigma/(1 - q)$ and variants $\sigma/(1 - q^r)^m$ with different combinations of exponents [10]. In general, the governing equations for a uniaxial stressed state are written in the form of a system of two equations:

$$\frac{d\varepsilon}{dt} = \frac{f(\sigma, T)}{\theta(q)},\quad \frac{dq}{dt} = \frac{\varphi(\sigma, T)}{Q(q)}. \tag{1}$$

The representation used for the second relation is based on the assumption that cumulative damage processes in the material are initially similar and, generally speaking, is a hypothesis requiring justification. The hypothesis of a common curve can

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physical significance or is specified by experimentally measurable quantities. It is important to note that Rabotnov's model does not impart any real physical significance to the damage parameter and leaves a certain leeway in the system of governing equations [7, 9]; it is impossible to determine the parameters of the equations independently from experimental data ([9], p. 366). Different authors eliminate this leeway in different ways, striving to achieve the best correspondence between the experimental data and calculated values [7, 10], but a unified procedure does not exist for determining the constants in the equations.

Kovpak [11] has analyzed numerous studies of the creep and long-term strength of metallic materials over a wide range of time periods; it follows from the results that the prefracture strain can increase initially and then decrease as the stress is increased, and vice versa, or it can decrease monotonically, etc. It is noted that the energy principle of damage is corroborated only in certain temperature and time intervals for certain types of materials. This fact essentially limits attempts to introduce a damage parameter endowed with a specific physical significance applicable to the description of deformation over a wide range of temperatures and times.

In this article, confining ourselves to the basic framework of Rabotnov's model and not ascribing any particular physical significance to the damage parameter, we set forth phenomenologically based arguments that permit the damage parameter to be related to experimentally measurable quantities and thus remove the leeway from the determination of the coefficients of the functional dependences in Eqs. (1).

## GOVERNING EQUATIONS

To simplify the presentation, we apply our discussion to unhardened materials, staying entirely within the bounds of the energy model of creep theory, where the power dissipation in creep $W = \dot{\varepsilon}_{ij}\sigma_{ij}$ is adopted as the measure of the intensity of the creep process [12]. We specialize the governing equations (1) to uniaxial creep and damage in energy terms:

$$\frac{dA}{dt} = \frac{f(\sigma, T)}{(1 - q)^{\mu}}, \quad \frac{dq}{dt} = \frac{\Phi(\sigma, T)}{(1 - q)^{k}} \quad (0 \leq q \leq 1), \tag{2}$$

where $A = \int_0^t W dt = \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij}$ is the work done in dissipation; the coefficients $\mu$ and k depend on the temperature T. Introducing the normalized time

$$\tau = (k + 1) \int_0^t \Phi(\sigma, T) dt \quad (0 \leq \tau \leq 1) \tag{3}$$

and integrating the second equation (2), we obtain

$$1 - \tau = (1 - q)^{k + 1}. \tag{4}$$

The substitution

$$1 - \omega = (1 - q)^{k - \mu + 1} \tag{5}$$

reduces the system of basic equations (2) to the form

$$\frac{dA}{dt} = \frac{f(\sigma, T)}{(1 - \omega)^{m}}, \quad \frac{d\omega}{dt} = \frac{\varphi(\sigma, T)}{(1 - \omega)^{m}}, \tag{6}$$

where $\varphi(\sigma, T) = (k - \mu + 1)\Phi(\sigma, T)$, $m = \mu/(k - \mu + 1)$, and

$$\omega = 1 - \left(1 - (m + 1)\int_0^t \varphi(\sigma, T) dt\right)^{1/(m + 1)}. \tag{7}$$

According to relations (4) and (5), we have the common-curve equation

$$(1 - \omega)^{m + 1} = 1 - \tau \ (0 \leq \omega \leq 1). \tag{8}$$

Allowing for the fact that $\omega$ is functionally related to the initial damage parameter q by Eq. (5) and varies within the same limits, the parameter $\omega$ can be estimated from the phenomenological principles of material damage. From now on we interpret $\omega$ as the damage parameter and write the governing equations for the creep process and long-term strength in the form (6) with identical softening coefficients m in both equations. The resulting system has one less coefficient than the basic system and no longer has leeway.

Thus, integrating the system (6) at constant stress, we find

$$\omega = 1 - [1 - (m + 1)\varphi(\sigma, T)t]^{1/(m+1)}, \quad A = \frac{f(\sigma, T)}{\varphi(\sigma, T)}\omega; \tag{9}$$

$$\omega = A/A_*, \quad A_* = f(\sigma, T)/\varphi(\sigma, T); \tag{10}$$

$$\tau = t/t_*, \quad t_* = \frac{1}{(m + 1)\varphi(\sigma, T)}. \tag{11}$$

In the case of uniaxial strain, therefore, the damage parameter is nothing other than the ratio of the instantaneous work of dissipation A to its value $A_*$ at the instant of fracture, i.e., the normalized work $\omega = A/A_*$, and $\tau$ is the normalized time, equal to the ratio of the given time to the fracture time $t_*$. Making use of the fact that $\omega = A/A_* \equiv \varepsilon/\varepsilon_*$ ($\varepsilon$ is the instantaneous strain, and $\varepsilon_*$ is the strain at the instant of fracture), we can relate the above-defined damage parameter to quantities that can be measured in a uniaxial experiment. Consequently, the consistency of the above-derived equations must be tested experimentally in the relative coordinates $\{\varepsilon/\varepsilon_*, t/t_*\}$.

For materials characterized by three creep stages, introducing the strength parameter $\alpha$ by analogy with [13], we write the governing equations in the form

$$\frac{dA}{dt} = \frac{f(\sigma, T)}{\omega^\alpha(1 - \omega^{\alpha+1})^m}, \quad \frac{d\omega}{dt} = \frac{\varphi(\sigma, T)}{\omega^\alpha(1 - \omega^{\alpha+1})^m}. \tag{12}$$

We now obtain a common-curve equation incorporating the above-introduced notation (3):

$$(1 - \omega^{\alpha+1})^{m+1} = 1 - \tau. \tag{13}$$

Here $\omega$ and $\tau$ have the form (10), (11).

The uniaxial equations (6) and (12) are generalized to a three-dimensionally stressed state in the usual way, where $\sigma$ is replaced by the equivalent stress $\sigma_e$ (a function of invariants of the stress tensor and the tensor of anisotropy of the creep properties), the only difference being that here the equivalent stresses are not the same in the creep and damage equations.

The consistency of the gradient flow law ($\dot{\varepsilon}_{kl} = \lambda\partial\sigma_e/\partial\sigma_{kl}$) using the equations in energy terms has been verified experimentally for many alloys [12], simplifying their practical utilization considerably. In application to the description of the process of creep and fracture of materials in energy terms, the system of equations for practical calculations is written in the form

$$\frac{dA}{dt} = \frac{f(\sigma_e, T)}{\omega^\alpha(1 - \omega^{\alpha - 1})^m}, \quad A = \int_0^t \sigma_{ij}\dot{\varepsilon}_{ij}dt,$$
$$\frac{d\omega}{dt} = \frac{\varphi(\sigma_{e*}, T)}{\omega^\alpha(1 - \omega^{\alpha+1})^m}, \quad \dot{\varepsilon}_{kl} = \lambda\frac{\partial\sigma_e}{\partial\sigma_{kl}}, \tag{14}$$

where $\sigma_e$ is the equivalent stress at constant power dissipation in the steady-state creep stage ($W_{min} = \sigma_{ij}\dot{\varepsilon}_{ij} = const$), and if this stage is omitted, $W_0 = const$ at the initial time ($t = 0$). The equivalent stress $\sigma_{e*}$ is defined as a combination of stressed states in which equivalent cumulative damage takes place under steady loading conditions and, accordingly, the time to fracture is identical.

Generally speaking, the system of equations (14) obtained in this way imposes rather stringent requirements on their domain of validity. Invoking the common-curve equations (8) and (13), we need to verify the similarity of the primary stress–strain curves of the material in the presence of creep up to fracture in terms of the damage of the material versus time. The process of creep and fracture must be determined entirely by the instantaneous values of the stress $\sigma$ and the material damage

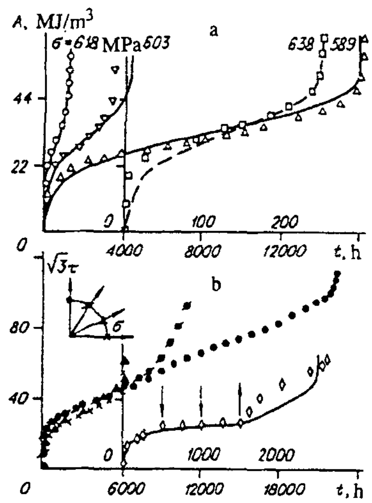


Fig. 1

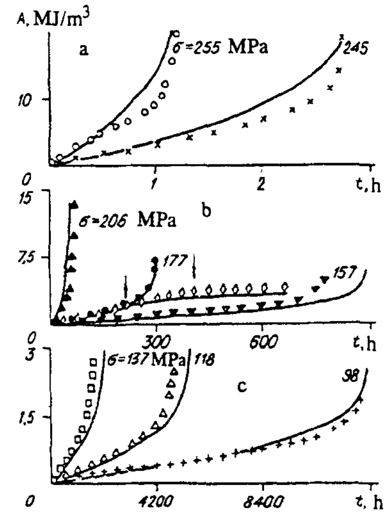


Fig. 2

ω independently of how the latter accumulates. The applicability of the proposed equations must be substantiated experimentally over a sufficiently broad range of temperatures and time.

## EXPERIMENTAL VERIFICATION OF THE SINGLE-CURVE HYPOTHESIS

We illustrate the foregoing considerations in the example of the creep and fracture process of two alloys under steady-state thermal and mechanical loading conditions: titanium alloy 3V at room temperature and aluminum alloy AK4-1T at a temperature of 200°C. Alloy 3V has a distinct hardening stage and substantial anisotropy of its creep properties (the time to fracture of samples cut transversely — relative to the rolling direction — from a slab of thickness 30 mm is 30 times the time to fracture of samples cut in the longitudinal direction for the same stress) [13]; it also exhibits an explicit dependence of the dissipation work in fracture on the type of stressed state and the sign of the applied load [14].

The light symbols in Fig. 1a represent the results of experiments on the tension of samples cut from slabs in the longitudinal direction in the form of A = A(t) diagrams for $\sigma$ = const (the values of $\sigma$ are indicated alongside the diagrams). Figure 1b shows the results [13, 14] of experiments on the tension, torsion, and combined tension and torsion of transversely cut samples for the same stress intensity ($\sigma_i$ = 637 MPa) plus an experiment on the tension of a longitudinally cut sample with the stress changed in steps from $\sigma_1$ = 608 MPa to $\sigma_2$ = 589 MPa to $\sigma_3$ = 559 MPa to $\sigma_4$ = 618 MPa (diamonds; the times at which the load changed are indicated by arrows). The work of dissipation in tension is about half that in torsion, whereas in tests with a fixed stressed state (Fig. 1a) the work of dissipation up to fracture is practically constant ($A_* \simeq 66.22$ MJ/m$^3$) over the entire investigated range of times both for longitudinal and for transverse cuts [13]. It has been shown [14] that a contour $\sigma_i$ = const (represented in the graph by the point symbols for the corresponding experiments with tension and combined tension and torsion) is an equivalent stress contour in the sense of equal power dissipation in the steady-state interval $\sigma_e = \sigma_i$, where the associative flow law is observed to be fully confirmed for this contour ($\dot{\varepsilon}_{kl} = \lambda \partial\sigma_e/\partial\sigma_{kl}$). The appreciable difference in the times to fracture in experiments with the same value of $\sigma = \sigma_i$ indicates that $\sigma_e \neq \sigma_{e*}$.

Figures 2-4 show experimental fatigue diagrams A = A(t) for the tension, torsion, and compression of samples cut transversely to the rolling direction from an alloy AK4-1T slab of thickness 64 mm at a temperature of 200°C and constant stress (the values of $\sigma_i$ indicated alongside the corresponding diagrams were held constant by the condition of incompressibility of the material in creep) in the range of time periods from a few hours to $10^4$ h. The samples were first subjected to artificial aging. The tension tests were performed with long cylindrical samples having a diameter 8 mm and a working length $l_0 \simeq 11\sqrt{F_0}$, shorter samples with $l_0 \simeq 6\sqrt{F_0}$ were used for the compression tests, and the samples for pure torsion had an outside diameter of 20 mm, an inside diameter of 18 mm, and a working length of 40 mm.

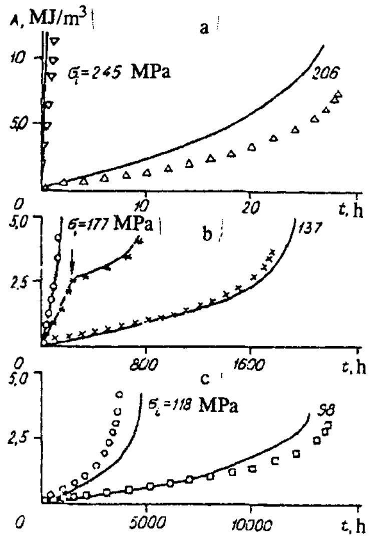


Fig. 3

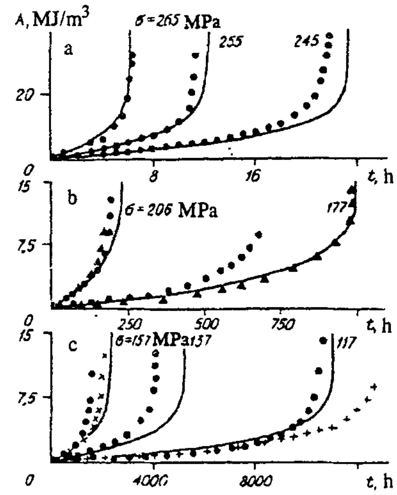


Fig. 4

It is evident from Figs. 2 and 3 that alloy AK4-1T differs from alloy 3V in that the work of dissipation in fracture depends strongly on the applied stress and varies by almost an order of magnitude in the investigated range of times. The diamond symbols in Fig. 2b represent the results of an experiment with a stepped variation of the stresses in tension from $\sigma_1$ = 177 MPa to $\sigma_2$ = 137 MPa to $\sigma_3$ = 118 MPa, and the stars in Fig. 3b represent the results of an analogous experiment in tension with a single load change (from $\sigma$ = 177 MPa to $\sigma$ = 137 MPa; the times of these changes are indicated by arrows).

In compression experiments (Fig. 4) the samples were not loaded to fracture; the experiments were stopped at strains of the order of 10–15%, when the sample began to bend (ductile-type buckling). Certain long-duration experiments were repeated (Figs. 4b and 4c). It follows from the results that the work of fracture $A_*$ depends on the sign of the applied load and is substantially greater in compression than in tension for equal duration of the process. An analysis of a few individual experiments in compression to fracture (alloy 3V and alloy ZhSK at a testing temperature of 800-850°C [15], among others) shows that the work of dissipation in fracture can exceed the value in tension by more than an order of magnitude for identical times.

Figure 5 (with the same notation as Figs. 1-3) shows the above-described experimental data (except in compression) converted to the normalized coordinates $\{\omega = A/A_*,\ \tau = t/t_*\}$. In the stepped-load experiments, to calculate the total quantities $\omega$ and $\tau$ in each kth load step, the increments of A and t are normalized to the values $A_{*k}$ and $t_{*k}$ for the corresponding steady-state experiment with the same stress values. Figure 5a shows the results of experiments on alloy 3V in the pure tension of samples cut longitudinally from a slab and in tension combined with torsion of samples cut in the transverse direction; Figs. 5b and 5c show the results of experiments on alloy AK4-1T in tension and in pure torsion, respectively.

Despite the rather complex behavior of the given alloys in the presence of creep and the explicit dependence of the work of dissipation in fracture on the magnitude and type of stressed state, the experimental points in the range from tension to torsion are clustered in a fairly tight corridor around a common curve in the normalized coordinates, both in steady and in stepped load-variation regimes. This corroborates the consistency of the representation of the damage kinetic equation in the form of an equation of state both for softened materials (8) and for materials with all three creep stages (13) over a broad range of time periods. A similar formal procedure for the plotting of experimental curves in normalized coordinates has been used previously [16, 17] in substantiating the equation for the fatigue curve over the entire range of few-cycle loads on the basis of the experimentally established similarity of cyclic creep curves in coordinates $\{\omega = \varepsilon/\varepsilon_*,\ \tau = N/N_*\}$ ($N_*$ is the number of cycles to fracture).

Consequently, by introducing the normalized coordinates $\{\omega = A/A_* = \varepsilon/\varepsilon_*,\ \tau = t/t_*\}$ for the processing of experimental data in steady loading regimes and having verified the consistency of the equation of state for the material damage parameter $\dot{\omega} = \varphi(\sigma)\psi(\omega)$ in these coordinates, we have succeeded in relating the damage parameter $\omega$ to the experimentally measurable quantities $\varepsilon$ and $\varepsilon_*$, thereby removing the leeway from the determination of the coefficients of the equations.

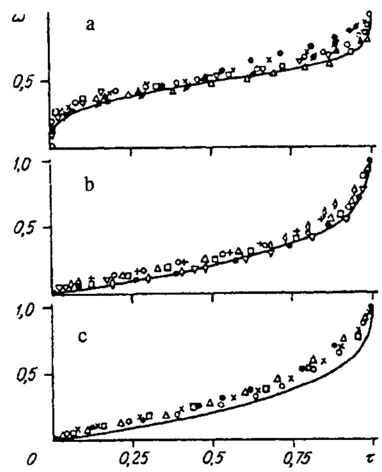


Fig. 5

## PROCEDURE FOR DETERMINING THE CHARACTERISTICS OF A MATERIAL; DISCUSSION OF THE RESULTS

Taking into account the foregoing substantiation of the damage equation in the form of an equation of state and the relationship of the damage parameter to the experimentally measurable quantities $\omega = \varepsilon/\varepsilon_*$, we now describe a unified procedure for determining the functional dependences and constants of the governing equations for creep and damage, as well as the corresponding approximation of the equivalent stresses in the example of alloy AK4-1T.

The procedure scarcely differs in form from that described in [12, 13]. The exponent m characterizes the softening of the material and is determined from a finite interval of the experimental common curve after the inflection point for materials with hardening. For softened materials ($\alpha = 0$) experimental data from the entire range of variation of the parameter $\omega$ are used in the determination of m. Thus, integrating the damage equation (6) for fixed values of the stresses $\sigma_k$ and temperature from certain instantaneous values of $\omega$ and $\tau$ to fracture $\omega = \tau = 1$, we obtain the relation

$$(1 - \omega)^{m+1} = (m + 1)\varphi(\sigma_k, T)(1 - \tau),$$

which represents the equation of a straight line in log–log coordinates $\{\ln(1 - \omega), \ln(1 - \tau)\}$, whose slope gives the coefficient m, which depends on the average of the values $m_k$ for several values of $\sigma = \sigma_k$.

The parameter $\alpha$, which characterizes the hardening, is determined according to data from the initial part of the common curve up to the point of transition to the steady-state creep stage. In this case, as in the deformation theory of hardening, we assume that $d\omega/dt = \varphi(\sigma, T)/\omega^\alpha$, whereupon, integrating the later from the zero state to the instantaneous values of $\omega$ and t and forming the logarithms, we obtain the straight-line equation

$$(\alpha + 1)\ln\omega = \ln[(\alpha + 1)\varphi(\sigma_k, T)] + \ln t.$$

The coefficient $\alpha$ is determined from its slope for several values of $\sigma_k$ (or directly from the averaged values of the common curve using the least-squares method).

The choice of the function $\varphi(\sigma)$ is determined by the long-term strength curve. For the investigated alloy AK4-1T the experimental points in log–log coordinates $\{\ln \sigma_i, \ln t_*\}$ have a monotonic dip from the ductile fracture region to the brittle fracture region with decreasing stresses and provide a fully satisfactory fit to straight lines in the coordinates $\{\sigma_i^2, \ln t_*\}$ (circles for tension, triangles for compression, and ×'s for pure torsion), so that the function $\varphi(\sigma)$ admits the natural approximation $\varphi(\sigma) = B_\omega(\exp\beta\sigma^2 - 1)$. The $-1$ is introduced to provide a correct description in the region of the deformation process at small stresses. Inasmuch as $\varphi(\sigma)$ is a monotonic function and the variations of the experimental values of $A_*(\sigma)$ tend to be monotonic [according to (10), $A_* = f(\sigma)/\varphi(\sigma)$], the function $f(\sigma)$ must have the same form $f(\sigma) = B_A(\exp\zeta\sigma^2 - 1)$. For

TABLE 1

| Test mode | $\alpha$ | $m$ | $B_A\ 10^9$, $\overline{\text{MJ}}/\text{m}^3\cdot\text{sec}$ | $B_\omega\ 10^9$, $\text{sec}^{-1}$ | $\zeta\ 10^4$, $\text{MPa}^{-2}$ | $\beta\cdot 10^4$, $\text{MPa}^{-2}$ |
|---|---|---|---|---|---|---|
| Tension | 0 | 2 | 3,0 | 2,014 | 2,09 | 1,611 |
| Torsion | 0 | 2 | 3,5 | 1,789 | 2,286 | 1,767 |
| Compression | 0 | 5 | 8,608 | 0,84575 | 1,515 | 1,2906 |

materials exhibiting a nonmonotonic dependence of the diffusion work (strains) in fracture on the stresses the experimental data can be approximated by the introduction of various functions f and $\varphi$ [18] and also on the basis of polynomials.

The coefficients of the equations $B_A$, $B_\omega$, $\zeta$, and $\beta$ (and, analogously, the coefficient B and the creep exponent n in the power-law relation) are evaluated by conventional methods from experimental data on the power dissipation and rate of accumulation of defects at the initial time $W = W_0$, $\omega = \omega_0$ or on the steady-state part of the creep curve if it exists.

In the compression experiments the samples were not loaded to fracture, making it impossible to plot an experimental $\omega - \tau$ curve, since $A_*$ and $t_*$ are not known for each $\sigma_k$ = const. Nonetheless, the given experimental curves can be used to determine the average time to fracture $t_*(\sigma_k)$ and hence to calculate $\tau = t/t_*$ to within the scatter of the experimental data (see Fig. 4). Transforming the first equation (6), we obtain the relation

$$W = f(\sigma_k)/(1 - \tau)^{m/(m+1)},$$

which represents a straight-line equation with slope m/(m + 1) in log−log coordinates. We thus find $m_k$ for each fixed $\sigma_k$. The average of the values $m_k$ gives the required m. In all other respects the evaluation of the coefficients of the equations is the same as above.

The solid curves in Figs. 2-4 and 5b represent the values of A(t) and $\omega(\tau)$ calculated from relation (14) for alloy AK4-1T with the characteristics shown in Table 1.

The dashed curve in the upper right inset of Fig. 6a represents the calculated contour of the equivalent stress $\sigma_e$ for the creep equation in energy terms, determined for $W_0 = 5.69\cdot 10^{-6}$ MJ/m$^3$·s = const. A planar stressed-state approximation is plotted on the basis of the functional relation

$$\sigma_e = B(1 + a\sin 3\xi + b\sin^2 3\xi)\sigma_i, \tag{15}$$

where $B = 1.0645$, $a = -4.764\cdot 10^{-3}$, and $b = -0.652\cdot 10^{-1}$ are constants obtained from the processing of three independent "certified" series of experiments in uniaxial creep at constant tensile, compressive, and shear (pure torsional) stresses [20]; $\xi$ is the angle of view of the stressed state. The solid curves in this figure represent two calculated contours of $\sigma_{e*}$ for fracture times $t_{*1}$ = 10 h and $t_{*2} = 10^4$ h. An approximation is also plotted from relation (15) with constants

$$B_* = 1.0302,\ a_* = -0.1154,\ b_* = -0.1446,$$

which are determined from the long-term strength curves in tension, compression, and torsion.

It is important to note that the experimental data on the long-term strength in tension, compression, and torsion in semilog coordinates $\{\sigma_i^2, \ln t_*\}$ are tightly clustered about a straight line. This means that the approximation of $\sigma_{e*}$ according to the relation (15) determined from "certified" experiments [20] can be used together with the standard quantities $\sigma_{e*}$ [21] for investigations in the planar stressed state. For materials with different properties in tension and compression in the general case of a combined stressed state the approximation of $\sigma_{e*}$ will most likely depend on all three invariants of the stress tensor, including the first [15, 22].

Figure 6b shows experimental A = A(t) diagrams for constant tensile (circles), compressive (triangles), and pure torsional (×'s) stresses of practically identical time to fracture. It is evident from these diagrams that alloy AK4-1T, unlike 3V, is not as strain-resistant in pure torsion as in tension. This relationship also emerges clearly from the long-term strength graph (Fig. 6a): The line for pure torsion is situated below the tension and compression lines. This also can be attributed to the anisotropy of the creep properties of the material.

For comparison Fig. 6c shows the pattern of cumulative damage in tension (solid curve) and in compression (dashed curve) in normalized coordinates. We see that the cumulative damage processes differ significantly in tension and compression.

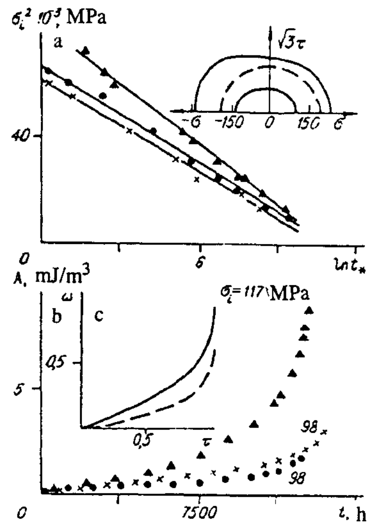

Fig. 6

This difference is attributable to the different creep resistances of the material, because the cumulative damage process is identical throughout the entire tension–torsion–compression domain in both steady and stepped loading for materials exhibiting identical behavior in tension and compression, for example, alloy D16T (Al—Cu—Mg) and steel 45 [19].

A similar conclusion as to a significant difference in the cumulative damage mechanisms in tension and compression has been drawn [23, 24] from dynamic elastic modulus measurements.

As an illustration, the solid and dashed curves in Figs. 1 and 5a represent the values of $A = A(t)$ and $\omega = \omega(\tau)$ calculated for alloy 3V with hardening in tension according to relations (14) using the power law

$$f(\sigma) = B_A(\sigma)^n, \quad \varphi(\sigma) = B_\omega(\sigma)^r.$$

The coefficients of the equations are calculated in accordance with data in [12]:

$$B_A = 1.4606 \cdot 10^{-151} \text{ MJ/(m}^3\cdot\text{MPa}^n\cdot\text{s)}, \quad B_\omega = 2.2054 \cdot 10^{-153} \text{ MPa}^{-r}\cdot\text{s}^{-1},$$
$$\alpha = 2.5, \quad m = 7, \quad n = r = 51.79.$$

Consequently, by reducing the general system of governing equations for creep and long-term strength to a system of equations with the same softening exponent in both equations for an arbitrary stress–strain state we have succeeded in relating the damage parameter to experimentally measurable quantities and thus eliminated the leeway in the determination of the evaluation of the constants of the equations.

The reduction of the experimental creep diagrams to a common curve in normalized coordinates $\{\omega = A/A_*, \tau = t/t_*\}$ for steady and stepped loading regimes applied to representative materials with a pronounced anisotropy of their creep properties, for which $A_*$ remains constant or decreases with decreasing stress, has enabled us to justify the representation of the kinetic damage equation in the form of an equation of state and to devise a unified procedure for finding the coefficients of the equations.

The fully satisfactory agreement of the calculated with the experimental values over a wide range of time periods supports the feasibility of using the governing equations of creep and damage with a single scalar damage parameter and identical hardening–softening exponents in both equations and also confirms the validity of the procedure for determining the characteristics of the material.

A more detailed analysis of the proposed relations calls for additional, specific goal-oriented experiments on creep and long-term strength in a combined stressed state, along with their certification testing over a wide range of temperatures and in a nonuniform stressed state.

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